Fourier Analysis Models and Their Application to River Flow's Prediction

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Abstract

The forecasting of hydrologic systems by using the date of the system is one of the most important advantages of time series analysis_(dynamic systems) in water science. By using time series analysis that is based on mathematical logics and statistical solutions and recent electronic solution methods, it is possible to evaluate the system's reactions in advance by using the systems past behavior characteristics. In most water resource systems design and operation studies, the periodic phenomena have been represented by Fourier functions. After Quimpo (1967), Fourier analysis has become a standard tool in any hydrologic study concerning periodicity because Fourier analysis and modeling present powerful tools to analyze different periodic events behavior. For analysis and design of water resource systems, it is sometimes useful to generate high- resolution synthetic river flows. Modeling and simulation of river flow time series is an important step in the planning and operational analysis of water resources systems. Thus, this paper compares three different Fourier based models in their capabilities and results. These three models are: Fourier PARMA models, Adapted Fourier analysis with kalman filter(AFAM), Fourier series ARIMA model(FSAM). These models test on "shahar-chayi" river flow. It is prospected that the results show the best way and its reliability.

Keywords*:* *Fourier analysis, Fourier transforms, river flow, kalman filtering.*

Introduction

Numerous factors contributed in environmental changes which ultimately resulted in creation of variable environs morphologically as well as applicably. Among the most vital factors is erosion which plays a crucial role in appearance and land use changes. Significant sorts of erosion include wind erosion, hydro-erosion as well as erosion due to human applications. Water with a surprising power is a key component in erosion and sedimentation riverbeds and coastal lines. Moreover, valleys and vast plains are formed due to water erosion which is mainly associated with water flow. To measure the water flow a quantity called "discharge" is applied. Thus, study of river erosion is carried out based on information about water flow (discharge measurement) in combination with geological properties. In this regard, investigation of collected data which provide the time series of river discharge is considered as a rational and applicable method to predict the future flow values. A natural river flow process has significant periodic behavior in mean, standard deviation, skewness, and serial dependence structure. This study aims to investigate and compare three conventional methods applied to measure flow discharge based on analysis of time series which include: 1.Fourier

Series ARIMA models (FSAM) 2.Adapted fourier analysis with "kalman" filter(AFAM) 3.Fourier PARMA models. These models test and compare on natural river flow.

Fourier series ARIMA models (FSAM)

Mostly, it is easier to show a function by a set of primary functions called as "base". So that it could be possible to illustrate the whole studied functions as linear composites of primary functions in base. Among the applicable function are (*sin*) as well as (*cos*) functions or mixed indices directly applicable to frequency analysis. In 1976, the studied model was applied for the first time by "Bloomfield" to analysis the time series in hydrology. "Afshar" and "Fahmi" (1996) provided a model to predict the rainfall in Iran by combining Fourier model with ARIMA models. Fourier model is a mathematical structure designed of Fourier equation in combination with "Markov model".

Fit pattern

Fit pattern is represented as follow:

$$X_{n,t} = \mu_t + \delta_t \xi_{n,t} \tag{1}$$

Where, μ_t and δ_t are average coefficient and standard deviation of harmonic series, respectively which are computed by following equation:

$$\mu_t = m_x + \sum_{j=1}^{\omega} (A_j Cos \frac{2\pi j}{\omega} t + B_j Sin \frac{2\pi j}{\omega} t) \tag{2}$$

Where, ω is series frequency period.

To compute the February series expansion coefficient (B_j and A_j), following equation is used:

$$A_j = \frac{2}{\omega} \sum_{j=1}^{\omega} m_j Cos \frac{2\pi j}{\omega} t \tag{3}$$

$$B_j = \frac{2}{\omega} \sum_{j=1}^{\omega} m_j Sin \frac{2\pi j}{\omega} t \tag{4}$$

In the above relation, m_t is average of inputs in a distinct interval time (month) during the statistical period and m_x is the total average of inputs.

Standard deviation of harmonic series, δ_t, could be measured as:

$$\delta_t = S_t + \sum_{j=1}^{\omega} (A'_j Cos \frac{2\pi j}{\omega} t + B'_j Sin \frac{2\pi j}{\omega} t) \tag{5}$$

Where,

$$A'_j = \frac{2}{\omega} \sum_{j=1}^{\omega} S_t Cos \frac{2\pi j}{\omega} t \tag{6}$$

$$B'_j = \frac{2}{\omega} \sum_{j=1}^{\omega} S_t Sin \frac{2\pi j}{\omega} t \tag{7}$$

S_t = standard deviation of inputs in a distinct interval time (month) during statistical period

Adapted Fourier analysis with "kalman" filter (AFAM)

AFAM model provided on the basis of a complicated mathematical structure has been simulated using statistic, mathematic as well as electronic sciences to predict the dynamic systems. "Zeki chen" (1980) attentively simulated discharge of flow in "Guta" and "Colombia" rivers and then predicted the future levels for the studied rivers. This model then called as kalman model:

$$(8)\ \hat{Y}(i|i-1) = \phi(i,i-1)\hat{Y}(i-1|i-1)$$

$$P(i|i-1) = \phi(i,i-1)P(i-1|i-1)\phi(i,i-1) + Q(i) \qquad (9)$$

$$(10)\ K(i) = P(i|i-1)H^T(i)\left[H(i).P(i|i-1)H^T(i) + R(i)\right]^{-1}$$

$$(11)\ \hat{Y}(i|i) = \hat{Y}(i|i-1) + K(i)\left[Z(i) - H(i)\hat{Y}(i|i-1)\right]$$

$$(12)\ P(i|i) = \left[I - K(i)H(i)\right]P(i|i-1)$$

There are three essential requirements in the model designed on the basis of kalman filter model:

1- state variable vector
2- Rule of transfer the state variable from one time to a further time
3- Primary state vector

In this model, Fourier equation is used as mode variable vector. In other words, the considered equation is the simulator function for real values. To design the model according to mentioned above analysis, by computation of $x(i) - x(i-1)$, following equation is achieved:

$$x(i) = x(i-1) + \sum_{k=1}^{m}\left(A_k(i-1)\left[\sin(\gamma ki) - \sin\{\gamma k(i-1)\}\right] + B_k(i-1)\left[\cos(\gamma ki) - \cos\{\gamma k(i-1)\}\right]\right) + \omega(i) \quad (13)$$

$$\alpha_k = \sin(\gamma ki) - \sin(\gamma k(i-1)), \beta_k = \cos(\gamma ki) - \cos(\gamma k(i-1)) \qquad (14)$$

$$x(i) = x(i-1) + \sum_{k=1}^{m}\left[A_k(i-1)\alpha_k + B_k(i-1)\beta_k\right] + \omega(i) \qquad (15)$$

$$\begin{bmatrix} x(i) \\ M(i) \\ A_1(i) \\ B_1(i) \\ A_2(i) \\ B_2(i) \\ \vdots \\ A_m(i) \\ B_m(i) \end{bmatrix} = \begin{bmatrix} 1 & 0 & \alpha_1 & \beta_1 & \alpha_2 & \beta_2 & \ldots & \alpha_m & \beta_m \\ 0 & 1 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 1 \end{bmatrix} . \begin{bmatrix} x(i-1) \\ M(i-1) \\ A_1(i-1) \\ B_1(i-1) \\ A_2(i-1) \\ B_2(i-1) \\ \vdots \\ A_m(i-1) \\ B_m(i-1) \end{bmatrix} + \begin{bmatrix} \omega(i) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Mathematical illustration of above matrix is:

$$(16)\ Y(i) = \phi(i,i-1).Y(i-1) + W(i)$$

Fourier-Parma Models

In the area of stochastic hydrology, standardizing or filtering is used to transform periodic time series to stationary series before fitting stationary stochastic models but standardizing or filtering of most river flow series will not yield stationary residuals due to periodic autocorrelations. in these cases, the resulting model is mis specified. To model such periodicity in autocorrelations, periodic autoregressive moving average (PARMA) models can be employed. in most cases, PARMA models have been applied to time series at a time scale of a months or more. However, when the number of periods is large (e.g, weekly data), PARMA models estimation of an exorbitant number of parameters, hereto for making PARMA modeling virtually impractical. the parsimony in these models is achieved by expressing the periodic model parameters in terns of their discrete fourier transforms. to find a parsimony model in time river discharge time series, "Tesfaye" e.t al(2007) found from their experience that it is prudent to initially fit a PARMAυ (1,1) model to the data. for more complicated PARMAυ(p,q) models, the periodic ARMA process $\{\widetilde{X}_t\}$ with period υ (denoted by PARMAυ(p,q))and the fourier series representation of the parameters $\phi_t(l), \theta_t(l)$ and σ_t are

$$X_t - \sum_{j=1}^{p} \phi_t(j) X_{t-j} = \varepsilon_t - \sum_{j=1}^{q} \theta_t(j) \varepsilon_{t-j} \tag{17}$$

$$\theta_t = C_{a0}(l) + \sum_{r=1}^{k} \left\{ C_{ar}(l) COS\left(\frac{2\pi rt}{\upsilon}\right) + S_{ar}(l) SIN\left(\frac{2\pi rt}{\upsilon}\right) \right\} \tag{18}$$

$$\varphi_t = C_{b0}(l) + \sum_{r=1}^{k} \left\{ C_{br}(l) COS\left(\frac{2\pi rt}{\upsilon}\right) + S_{br}(l) SIN\left(\frac{2\pi rt}{\upsilon}\right) \right\} \tag{19}$$

$$\sigma_t = C_{d0} + \sum_{r=1}^{k} \left\{ C_{dr} COS\left(\frac{2\pi rt}{\pi}\right) + S_{dr} SIN\left(\frac{2\pi rt}{\pi}\right) \right\} \tag{20}$$

Where, $X_t = \widetilde{X}_t - \mu_t$ and ε_t= sequence of random variables with mean zero and scale σ_t such that $\{\delta_t = \sigma_t^{-1} \varepsilon_t\}$ is independent and identically distributed. k= total number of harmonics, which is equal to $\upsilon/2$ or $\upsilon - 1/2$ depending on whether υ is even or odd, respectively. the validation of a time series model is tantamount to the application of diagnostic checks to the model residuals to see if they resemble white-noise. to test the white-noise null hypothesis the ljung-box test has used. if the null hypothesis of white-noise residuals is not rejected and if the autocorrelation and partial autocorrelation function of the residuals show no evidence of serial correlation, then we judge the model to adequate.

Conclusion

In this study, using observation information about "Shahr-chayi" River along with Fourier, Fourier kalman as well as Fourier PARMA models, time series of river flow has been simulated and applied to predict the future flow. Results showed that every three models were strongly applicable in modeling of dynamic time series. Meanwhile, FSAM model due to application of ARIMA model and elasticity of Fourier equation has been located in a higher position rather than ARIMA linear models. In addition, AFAM model was remarkably able to rapidly reduce the errors and improve the results due to application of kalman filter. Also, PARMA (1, 1) model showed a significant accordance with observation information due to application of lower functions. Overall, increasing statistic regarding time series of monthly flows highly develop the applicability of these models in short term predictions in order to be applied in black box modeling.

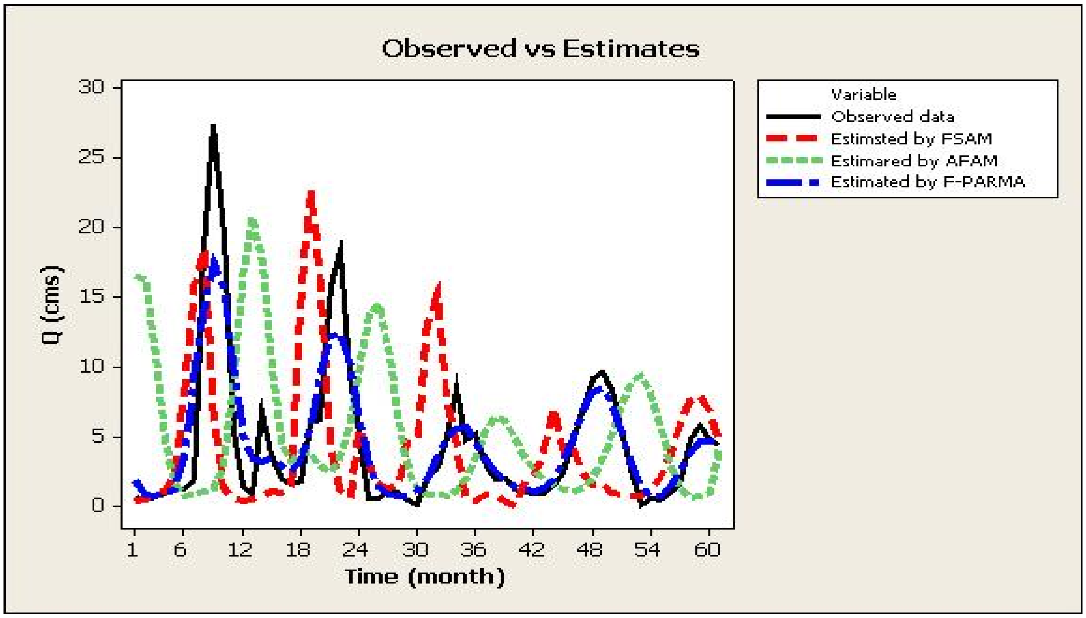

figure 1: Plot of observed and simulated 60 monthly river flows Data for the "Shahar-chayi" river – Urmia-IRAN

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